

## Errata and Updates for ASM Exam LTAM (First Edition Fourth Printing) Sorted by Date

For Practice Exam 4:B2(c), make the correction for 7/22/20.

For Practice Exam 7:B4, make the correction for 7/22/20.

For Practice Exam 7:B6, make the correction for 7/22/20.

For Practice Exam 9:B4, make the correction for 7/22/20.

Practice Exam 7:A8 is defective in that none of the five answer choices is correct.

For Practice Exam 12A10, make the correction for 6/23/20.

For Practice Exam 12B2(b)(ii), make the correction for 8/30/19.

For Practice Exam 13:A9, make the correction for 7/22/20. On page 1590, in the solution to question 2(b), change 174,349.35 to 177,686.10 on the last two lines, once apiece on each line. In the solution to question 2(c), on the last line, change 174,349.35 to 177,686.10 and change 11,325.02 to 11,450.14.

[8/23/2020] On page 904, make the following corrections to the solution to exercise 47.2:

- On the fifth line, change  $e^{-0.0105t}$  to  $e^{-0.105t}$ .
- On the ninth and tenth lines, change  $e^{-1.305}$  to  $e^{-1.2}$ .
- On the eleventh line, change 0.382728 to 0.425100 and change 0.125572 to 0.072607.
- On the last line, change  $A_1 + A_2$  to  $\frac{50,000(A_1 + A_2)}{8.815557}$
- On the last line, change 0.125572 to 0.072607 and change the final answer to **12,857.56**.

[8/2/2020] On page 1718, replace the last 3 lines of the solution to question 6(c) with

$$e^{-0.06(16)}({}_{16}P^{00} + {}_{16}P^{02}) = e^{-0.96} \left( e^{-0.72(16)} + \frac{3}{7}(e^{-0.02(16)} - e^{-0.72(16)}) \right) = 0.293089$$

The sum of these amounts multiplied by 100,000 is

$$100,000(0.025641 + 0.066364 + 0.293089) = \mathbf{38,509.37}$$

[7/23/2020] On page 1691, in the solution to question 1(a), change the final answer to 65,685.58.

[7/23/2020] On page 1694, in the solution to question 4(b), on the third line, change 0.02(64.9) to 0.02(29.9).

In the solution to question 4(c), change the last 3 lines to

$$\begin{aligned} &= 0.02e^{-0.5} \int_0^{10} t e^{-0.0075t^2} dt \\ &= -\frac{0.02e^{-0.5}}{0.015} e^{-0.0075t^2} \Big|_0^{10} \\ &= -\frac{0.02e^{-0.5}}{0.015} (e^{-0.75} - 1) = \mathbf{0.426701} \end{aligned}$$

In the solution to question 4(d), on the last line, change 0.353206 to 0.426701 and change 0.360289 to 0.286794.

[7/23/2020] On page 1699, in the solution to question 9, change the answer key to **(A)**. Also change the answer key on page 1697 accordingly.

[7/23/2020] On page 1706, in the solution to question 6(c), on the second-to-last line, change  $s_{329.72}$  to  $2329.72$ .

[7/23/2020] On page 1714, replace the last 3 lines of the solution to question 2(c) with

Linearly interpolating between  $l_{94}$  and  $l_{95}^-$ ,

$$\frac{25,094.3 - 24,834.6}{25,094.3 - 21,569.9} = 0.07369$$

The third quartile of survival time is **54.07369**.

[7/22/2020] On page 1145, in the solution to exercise 60.39, on the first line, change  ${}_{10|10}q_{30:40}$  to  ${}_{10|10}q_{\overline{30:40}}$ .

[7/22/2020] On page 1284, in the solution to exercise 68.25, on the first line, replace “eq:VarSnextrap” with “(68.6)”.

[7/22/2020] On page 1289, on line 5 of the second paragraph, change “survival survivals” to “survival probabilities”.

[7/22/2020] On page 1337, replace the solution to exercise 70.18 with

We need to calculate the probability of retirement at each age. From the wording of the retirement probabilities, we are given independent associated single decrements that happen one after the other. So we calculate probability of survivors to age 63, multiply that by 10% to get retiree count, subtract that, and then do the same at age 64. In the following table, all  $l_x$  and  $q_x$  are from the Standard Ultimate Life Table, and “non-retirement probability” is the probability of survival to the indicated age and not retiring at that age.

Age $x$	Survivorship probability ${}_{x-45}p_{45}^{(\tau)}$	Retirement probability ${}_{x-45}q_{45}^{(r)}$	Non-retirement probability
63	$\frac{l_{63}}{l_{45}} = \frac{95,534.4}{99,033.9} = 0.964664$	$0.1(0.964664) = 0.096466$	$0.964664 - 0.096466 = 0.868197$
64	$0.868197p_{63} = 0.868197(1 - 0.004730) = 0.864091$	$0.1(0.864091) = 0.086409$	$0.864091 - 0.086409 = 0.777682$
65	$0.777682p_{64} = 0.777682(1 - 0.005288) = 0.773569$	$0.773569$	

The annual payment under the pension is  $(10)(0.02)(100,000) = 20,000$  before reduction for early retirement.

We use the values of  $\ddot{a}_x$ ,  $x = 63, 64, 65$  found in the Standard Ultimate Life Table for the retirement annuity values. The following table lists the factors we use:

Age	Reduction Factor	Retirement Probability	$\ddot{a}_x$	Discount Factor
63	0.8	0.086820	14.1151	$1/1.05^{18}$
64	0.9	0.077768	13.8363	$1/1.05^{19}$
65	1.0	0.773569	13.5498	$1/1.05^{20}$

The expected present value of the retirement benefit is

$$20,000 \left( \frac{(0.8)(0.096466)(14.1151)}{1.05^{18}} + \frac{(0.9)(0.086409)(13.8363)}{1.05^{19}} + \frac{(0.773569)(13.5498)}{1.05^{20}} \right) = \mathbf{96,577.88}$$

[7/22/2020] On page 1347, in exercise 71.3, replace (iii) with

The expected amount of money that the plan will pay during the year for exit benefits is 10,105.

[7/22/2020] On page 1407, in exercise 75.8, change  $0.002k$  to  $0.002j$ .

[7/22/2020] On page 1479, in question 2(c), change 26 to 16.

[7/22/2020] On page 1497, in question 20(i), change “years” to “year”.

[7/22/2020] On page 1508, in question 4(b), on the third line, change “1.3” to “1.2”.

[7/22/2020] On page 1509, in question 6(b), delete “,using  $i=0.06$ ”. In question 6(f), change “It” to “it”.

[7/22/2020] On page 1528, in question 4(e), change “(c)” to “(d)”.

[7/22/2020] On page 1558, in the schematic diagram, change “4” in the Dead box to “3”.

[7/22/2020] On page 1563, in question 9, change the headings in the table from  $d_j, x_j, u_j$  to  $n_j, d_j, w_j$  respectively.

[7/22/2020] On page 1607, on the last line of the page, change “MLC” to “LTAM”.

[7/22/2020] On page 1613, in the solution to question 2(a), on the first line, change  $-26\ddot{a}_{55}$  to  $-16\ddot{a}_{55}$ . In the solution to question 2(b), change the last two lines to

$$258.2065 - 15.57760G = 0$$

so  $G = \boxed{16.5755}$ .

[7/22/2020] On page 1625, at the end of the solution to question 2(a), delete the stray “S”.

[7/22/2020] On page 1626, in the solution to question 3(b), change the last two lines to

$$1000(8.10782) + (x - 1000)(8.10782 - 7.9555) = 7955.5 + 0.1523x$$

The total EPV is  $14,321.1 + 2.7562x$ . Equating to 17,599.7, we get  $x = \boxed{1189.5}$ .

[7/22/2020] On page 1627, in the solution to question 4(c), remove the framed shaded box from 9.9409 and add the following line after that line:

$$10,000_2\ddot{a}_{66} = \boxed{99,409}$$

[7/22/2020] On page 1653, replace the solution to question 6(b) with

The EPV at age 65 of the life annuity-due is  $137,315.6\ddot{a}_{65} = 137,315.6(13.5498) = 1,860,600$ . Discounting to age 35,

$$1,860,600_{30}E_{35} = 1,358,999(0.61069)(0.35994) = \boxed{408,982}$$

[7/22/2020] On page 1658, replace the solution to question 19 with

We need to calculate the present value of exit benefits. Three times salary is 180,000.

$$\frac{180,000q_{40}}{(1+i)^{0.5}} = \frac{180,000(0.000527)}{1.05^{0.5}} = 92.57$$

The normal contribution is

$$62,537vp_{40} + 92.57 - 55,647 = 62,537\left(\frac{1 - 0.000527}{1.05}\right) + 92.57 - 55,647 = \boxed{3973} \quad (D)$$

[7/22/2020] On page 1661, in the solution to question 5(a) and 5(b), change 3195.1 to 319.51 in the two places it appears.

[7/22/2020] On page 1667, in the solution to question 14, replace the answer key (D) with (C). Also correct the answer key on page 1664.

[7/22/2020] On page 1668, in the solution to question 17, replace the answer key (E) with (B). Also correct the answer key on page 1664.

[7/22/2020] On page 1685, in the solution to question 6:

- In part (a), on the first displayed line, change 4,354.70 to 4,152.44. On the second displayed line, change 17,295.00 to 16,491.72. Change the final answer from 21,649.70 to 20,644.16.
- In part (c), on the third line from the end, change  $0.03(120,178.6)$  to  $0.03(20,178.6)$ .
- Replace the solution to part (d) with  
As calculated in part (b), annual benefit accrual with projected career average is 2605.36. With 35 years of service, the annual pension is  $25(2605.36) = 65,134$ . Final salary is  $100,000(1.025^{19}) = 159,865$ . The replacement ratio is  $65,134/159,865 = \boxed{0.4074}$ .

[7/22/2020] On page 1689, in the solution to question 13, on the second and seventh lines, change 0.37135 to 0.36135.

[7/21/2020] On page 464, in the solution to exercise 23.19, 4 lines before the end, change  $\ddot{a}_{67:\overline{2}|}$  to  $\ddot{a}_{67:\overline{3}|}$ .

[7/21/2020] On page 508, in the solution to exercise 26.14, on the second line, change  $-0.2$  to  $-0.02$ . On the third line, change 0.2 to 0.02.

[7/21/2020] On page 595, on the first line of Example 32A, add “insurance” after “endowment”.

[7/21/2020] On page 601, on the second-to-last line of exercise 32.17, add “insurance” after “endowment”.

[7/21/2020] On page 648, in the solution to exercise 35.15, on the last line, change (AD) to (A).

[7/21/2020] On page 713, in the solution to exercise 39.1, three lines before the end, change the numerator of the fraction from  $0.864048 - 746876^2$  to  $0.746876 - 0.864048^2$ . And change the final answer from 0.000259 to 0.00259.

[7/21/2020] On page 745, in exercise 40.60(b), on the second line, change “annual net premium” to “net level annual premium”.

[7/21/2020] On page 758, in the solution to exercise 40.41, replace the last line with

$${}_{19}V = \frac{176.4470 + 5.288}{1.05} - 6.78 = \boxed{166.30}$$

[7/21/2020] On page 776, in the solution to exercise 41.15, replace the last displayed line with

$${}_{15}V^{\text{mod}} = 235.24 - (7.0638)(4.5268) - (6.5586)(11.5331) = \boxed{127.62}$$

[7/21/2020] On page 856, in exercise 46.1, all annuities should be continuous. Therefore the question should read  
A Markov chain has three states: Healthy (0), Sick (1), and Dead (2). Movement between states follows the Sickness-Death model and  $i = 0.05$ .

A 10-year deferred continuous whole life annuity on a healthy person age 55 pays 10,000 per year starting at age 65 as long as the annuitant is alive. Premiums are payable continuously for 10 years as long as the annuitant is healthy.

Calculate the annual net premium.

- [7/21/2020] On page 868, in the solution to exercise 46.1, all annuities should be continuous. Replace every  $\ddot{a}$  with  $\bar{a}$ . There are six  $\ddot{a}$ s.
- [7/21/2020] On page 884, after the third line of the solution to Example 47E, add:  
 Reentry to state 2 is possible by first going to state 1 and then to state 0, so  $\bar{A}_{x+t}^{22}$ , the EPV of an insurance that pays 1 upon reentry to state 2, is not 0. However,  $\mu_{x+t}^{02} = 0$ , so this does not affect the final formula.  
 On the fourth line, change  $\mu_{x+t}^{02}(1 - \bar{A}_{x+t}^{02})$  to  $\mu_{x+t}^{02}(1 + \bar{A}_{x+t}^{22} - \bar{A}_{x+t}^{02})$ .
- [7/21/2020] On page 885, on the third line of Example 47F, change “if the accelerated death benefit was not used,” to “if the insured was never chronically ill;”.
- [7/21/2020] On page 970, in exercise 50.1, on the second line, change  ${}_4p_{50}^1$  to  ${}_4p_{50}^{(1)}$ . On the third line, change  ${}_4p_{50}^2$  to  ${}_4p_{50}^{(2)}$ . In exercise 50.2, on the fourth line, change  $p_x^2$  to  $p_x^{(2)}$ .
- [7/21/2020] On page 1000, on the second line of the solution to Example 52D, replace the clause after the semicolon with  
 in the absence of decrement 1, 30% of the population at time 0 is decremented at time 1 by decrement 2.
- [7/21/2020] On page 1065, in the solution to exercise 56.16, on the second line, change  ${}_5p_{90:90}$  to  ${}_5q_{90:90}$ .
- [7/21/2020] On page 1095, in exercise 58.16(ii), change  $i = 0.05$  to  $\mu = 0.05$ .
- [7/20/2020] On page 1, on the first line of the first bullet, change “emphsurvival” to “survival”.
- [7/20/2020] On page 97, in the solution to exercise 6.16, on the first line, change “between 21” to “between 20”.
- [7/20/2020] On page 101, on the last line of the solution to exercise 6.30, delete “o5 = ”.
- [7/20/2020] On page 192, in exercise 11.6, change (i) to  $\mu_{x+t} = \alpha/(100 - (x + t))$ ,  $t \geq 0$ .
- [7/20/2020] On page 227, in the solution to exercise 12.18, split the last line into two, putting “Answer ...” on the second line.
- [7/20/2020] On page 302, in the solution to exercise 15.16, on the second line, change the exponent on  $\left(\frac{40-t}{40}\right)$  from 0.5 to 0.3.
- [7/19/2020] On page 919, two lines below equation (48.6), change “ $t^{\text{th}}$  year” to “ $k^{\text{th}}$  year”.
- [7/19/2020] On page 991, in the solution to exercise 51.6, on the first line, change  ${}_3p_0^{(j)}$  to  ${}_2p_0^{(j)}$ .
- [7/19/2020] On page 1516, in question B1 (iii), change 10,000,000 to 100,000.
- [7/19/2020] On pages 1658–1659, replace the solution to question 1(b)–(c) with

- (b)  ${}_2p_{40}$  in Standard Ultimate Life Table is  $l_{42}/l_{40} = 99,229.8/99,338.3$ . In your table,  $\mu$  is 0.0001 lower, so

$${}_2p_{40} = \left(\frac{99,229.8}{99,338.3}\right)e^{0.0001(2)} = 0.999108$$

Select mortality is that probability raised to the 0.8, or  $0.999108^{0.8} = 0.999286$ . Working backwards from  $l_{42}$ , we get  $l_{[40]} = 99,448.3/0.999286 = \boxed{99,519.4}$ .

- (c) It’s easier to work with annuity factors than directly with premium factors. Note that  $d = 1 - v = 0.05$ .

$$\ddot{a}_{[41]+1} = \frac{1}{P_{[41]+1} + d} = \frac{1}{0.003 + 0.05} = 18.86792$$

Do recursion to 43, then to 42, then to  $[40]+1$ .

$$p_{[41]+1} = (p_{42}^{\text{SULT}} e^{0.0005})^{0.8} = ((1 - 0.000608)e^{0.0001})^{0.8} = 0.999594$$

$$\ddot{a}_{43} = \frac{\ddot{a}_{[41]+1} - 1}{v p_{[41]+1}} = \frac{17.86792}{0.95(0.999594)} = 18.81599$$

$$p_{42} = p_{42}^{\text{SULT}} e^{0.0001} = (1 - 0.000608)e^{0.0001} = 0.999492$$

$$\ddot{a}_{42} = 1 + v p_{42} \ddot{a}_{43} = 1 + (0.95)(0.999492)(18.81599) = 18.86611$$

$$p_{[40]+1} = (p_{41}^{\text{SULT}} e^{0.0001})^{0.8} = ((1 - 0.000565)e^{0.0001})^{0.8} = 0.999628$$

$$\ddot{a}_{[40]+1} = 1 + (0.95)(0.999628)(18.86611) = 18.91613$$

$$P_{[40]+1} = \frac{1}{\ddot{a}_{[40]+1}} - d = \frac{1}{18.91613} - 0.05 = \boxed{0.002865}$$

[7/9/2020] On page 1365, in the solution to exercise 72.5, on the third line, the fraction should be inverted; change  $\frac{1.04}{1.005(1.05)}$  to  $\frac{1.005(1.05)}{1.04}$ .

[7/6/2020] On page 1346, on the second line after Table 71.1, delete “to” between “may” and “use”.

[7/6/2020] On page 1348, in exercise 71.5, add the following statement after (v):

(vi) There are no exits from the pension plan within the first 10 years of service.

[6/26/2020] On page 1274, exercise 68.12 is missing answer choices. Add the following line at the end:

(A) 10                      (B) 12                      (C) 15                      (D) 17                      (E) 18

[6/23/2020] On page 1165, in the solution to exercise 61.1, remove the comma after  $v^5$  on the first line. On the fifth line, change  $a_{30:40:\overline{10}}$  to  $a_{30:40:5}$  and  ${}_{10}E_{30:40} a_{35:45}$  to  ${}_5E_{30:40} a_{35:45}$ .

[6/23/2020] On page 1552, in question 10, on the second line from the end, change  ${}_0V = 2711.75$  to  ${}_0V = 0$ .

[6/23/2020] On page 1699, in the solution to question 10, delete the second sentence “The initial reserve...”.

[6/22/2020] On page 1126, in exercise 60.23, on the first line, add “with” between “Markov chain” and “the”.

[6/22/2020] On page 1145, in the solution to exercise 60.34, on the second line, add at the end of the sentence “if (y) dies first”.

[6/19/2020] On page 635, in the solution to exercise 34.3, on the fifth line, change 2.236 to 2.326.

[6/5/2020] On page 897, in exercise 47.12, on the fourth line, add “if alive at that time” after “subsequent years”.

[6/5/2020] On page 897, in exercise 47.13, 2 lines from the end, delete “of”.

[6/5/2020] On page 907, in the solution to exercise 47.12, on the seventh line, delete “but we don’t pay 10,000... at the end of the year.”

[6/4/2020] On page 896, in exercise 47.10, in statement (i), delete  $\bar{A}_{57}^{01} = 0.23645$ . In statement (ii), delete  $\bar{A}_{67}^{01} = 0.33642$ .

[6/4/2020] On page 906, replace the solution to exercise 47.10 with

We need  $\bar{A}_x^{01}$  and  $\bar{A}_x^{02}$  for  $x = 57, 67$ .  $\bar{a}_x^{33}$  is the present value of a perpetuity, so it is  $1/0.05 = 20$ . Then  $\bar{a}_x^{03}$  is the EPV of an insurance that pays 20 if and when reaching state 3, so it equals  $20\bar{A}_x^{03}$ . Similarly,  $\bar{a}_x^{02} = 20\bar{A}_x^{02}$ . Also,  $\bar{a}_x^{01} + \bar{a}_x^{03}$  is the EPV of a perpetuity paying 1 per year if and when reaching state 1, so  $\bar{a}_x^{01} + \bar{a}_x^{03} = 20\bar{A}^{01}$ . And  $\bar{a}_x^{00} + \bar{a}_x^{01} + \bar{a}_x^{02} + \bar{a}_x^{03} = 20$  since it is the present value of a perpetuity. Putting this

all together:

$$\begin{aligned}\bar{A}_x^{01} &= \frac{\bar{a}_x^{01} + 20\bar{A}_x^{03}}{20} = \frac{\bar{a}_x^{01}}{20} + \bar{A}_x^{03} \\ \bar{A}_{57}^{01} &= \frac{2.18}{20} + 0.16344 = 0.27244 \\ \bar{A}_{67}^{01} &= \frac{1.59}{20} + 0.20021 = 0.27971 \\ \bar{A}_x^{02} &= \frac{20 - \bar{a}_x^{00} - \bar{a}_x^{01} - 20A_x^{03}}{20} \\ \bar{A}_{57}^{02} &= \frac{20 - 11.55 - 2.18 - 20(0.16344)}{20} = 0.15006 \\ \bar{A}_{67}^{02} &= \frac{20 - 8.21 - 1.59 - 20(0.20021)}{20} = 0.30979\end{aligned}$$

The net premium is

$$\begin{aligned}P &= \frac{70,000\bar{A}_{57}^{01} + 100,000\bar{A}_{57}^{02} + 30,000\bar{A}_{57}^{03}}{\bar{a}_{57}^{00}} \\ &= \frac{70,000(0.27244) + 100,000(0.15006) + 30,000(0.16344)}{11.55} = 3374.892\end{aligned}$$

The reserve at time 10 is

$$\begin{aligned}_{10}V^{(0)} &= 70,000\bar{A}_{67}^{01} + 100,000\bar{A}_{67}^{02} + 30,000\bar{A}_{67}^{03} - P\bar{a}_{67}^{00} \\ &= 70,000(0.27971) + 100,000(0.30979) + 30,000(0.20021) - 3374.892(8.21) = \boxed{28,857.14}\end{aligned}$$

[6/2/2020] On page 884, replace the second sentence of the solution to Example 47E, which starts with “For  $\bar{A}_{x+10}^{02}$ ”, with

The annual cost of care is not included in the equation, since a whole life insurance does not pay any annuity benefits.

[6/2/2020] On page 885, on the third line, change  $\bar{a}_{x+10}^{01}$  to  $\bar{a}_{x+10}^{10}$ .

[6/1/2020] On page 854, in the solution to Example 46M, in part 2, change the final answer from 18,282.4 to 18,122.4. On the last line of the solution, change 18,282.4 to 18,122.4 and change 7,786 to 7,946.

[5/18/2020] On page 707, in the solution to Example 39C, on the fourth line, change  $E$  to  $e$ .

[5/17/2020] On page 654, in the solution to Example 36E part 2, on the fourth line, change 0.88392 to 0.77382.

[5/13/2020] On page 632, one line above Example 34B, change 14.249 to 14.239.

[5/10/2020] On page 540, replace the fifth line before the end of the solution to exercise 28.2 with

$$\alpha(12) = 1.00020 \quad \beta(12) = 0.46651$$

To be consistent, three lines before the end of the solution, replace 1.000197 with 1.00020 and 0.466506 with 0.46651, although the more precise values are correct.

[4/25/2020] On page 293, three lines after the solution to Example 15J, change “ $x > n$ ” to “ $t > n$ ” and change “between  $x$  and  $n$ ” to “between  $t$  and  $n$ ”.

[4/25/2020] On page 618, on the sixth and seventh lines of the solution to Example 33G, change 58.0944 to 47.3396 (once on each line).

[4/23/2020] On page 235, in the solution to exercise 12.49, on the second line, change the first numerator 315 to 0.315. Replace the last two lines with

$$F_1 = 448.253(1.07) - 1000 = -520.370$$

$$F_2 = -520.370(1.069) - 500 = -1056.275$$

The excess of expected over actual is **1056.275**.

[4/22/2020] On page 209, on the first line of Example 12H, put a space between “that” and “ $q_{45}$ ”.

[4/20/2020] On page 188, in the solution to exercise 10.17, on the third line from the end, change  $p(85, 2017)$  to  $p(80, 2017)$ .

[4/19/2020] On page 732, in the box before exercise 40.10, replace the bottom two lines with

$$1000P_{68} = 34.12 \qquad \qquad \qquad \ddot{a}_{68:\overline{20}|} = 12.294465$$

$$1000A_{68:\overline{20}|} = 527.14 \qquad \qquad \qquad \ddot{a}_{85:\overline{3}|} = 2.721092$$

[4/19/2020] On page 751, replace the solutions to exercises 40.10–40.13 with the following:

40.10 Calculate this prospectively.

$${}_{17}V = 1000(A_{85:\overline{3}|} - P_{68}\ddot{a}_{85:\overline{3}|})$$

$$A_{85:\overline{3}|} = 1 - d\ddot{a}_{85:\overline{3}|} = 1 - \left(\frac{0.04}{1.04}\right)(2.721092) = 0.895343$$

$${}_{17}V = 1000(0.895343) - 34.12(2.721092) = \mathbf{802.50}$$

40.11 Calculate this recursively. Even though we are using a 4% interest rate, we still refer to the Standard Ultimate Life Table to obtain mortality rates. From the Standard Ultimate Life Table, we use  $q_{68} = 0.008297$  for the first recursion and  $q_{69} = 0.009294$  for the second recursion.

$${}_1V = \frac{(34.12)(1.04) - 8.297}{1 - 0.008297} = 27.4153$$

$${}_2V = \frac{(27.4153 + 34.12)(1.04) - 9.294}{1 - 0.009294} = \mathbf{55.22}$$

40.12 From the Standard Ultimate Life Table, we use  $q_{73} = 0.014664$  to go from  ${}_5V$  to  ${}_6V$ .

$$\frac{(140.42 + \pi_5)(1.04) - 0.014664(1000)}{1 - 0.014664} = 190$$

$$\pi_5 = \frac{190(1 - 0.014664) + 14.664}{1.04} - 140.42 = \mathbf{53.69}$$

40.13 The expected present value of the premiums is  $34.12(12.294465)$  plus the expected present value of the extra  $270 - 34.12$ , or  ${}_5E_{68}(270 - 34.12)$ , which we compute using the Standard Ultimate Life Table. We compute  ${}_5E_{68}$  by taking the value from our tables (0.74305) and multiplying it by  $(1.05/1.04)^5$  to convert it from 5% to 4%:

$${}_5E_{68} = 0.74305 \left(\frac{1.05}{1.04}\right)^5 = 0.77947$$

$$\text{PVPrem} = (34.12)(12.294465) + 0.77947(270 - 34.12) = 603.3475$$



The EPV of the benefits must match this. The EPV of the standard benefits of 1000 is  $1000A_{68:\overline{20}|} = 527.14$ . The excess of the present value of premiums over this,  $603.3475 - 527.14 = 76.2075$ , must be the EPV of the extra death benefit, which is  $({}_{15|}q_{68}/1.04^{16})(b_{16} - 1000)$ . From the Standard Ultimate Life Table,

$${}_{15|}q_{68} = \frac{l_{83} - l_{84}}{l_{68}} = \frac{67,614.6 - 64,506.5}{92,706.1} = 0.033526$$

Then

$$\begin{aligned} \frac{{}_{15|}q_{68}}{1.04^{16}}(b_{16} - 1000) &= 76.2075 \\ b_{16} - 1000 &= \frac{76.2075(1.04^{16})}{0.033526} = 4257.42 \\ b_{16} &= \boxed{5257.42} \end{aligned}$$

[4/18/2020] On page 403, on the seventh line from the bottom, delete  $v^n$ . Replace the sentence beginning “let  $I = 0$  if  $T_x \leq n$ ” with

But consider the *conditional* variable  $Y | T_x > n$ . Now that survival to time  $n$  is guaranteed, this variable is  $v^n \bar{a}_{\overline{T_{x+n}|}}$ . And of course the conditional variable  $Y | T_x \leq n$  is the constant 0. For convenience, let  $I = 0$  if  $T_x \leq n$  and 1 otherwise.

On the third line from the bottom, replace  ${}^2\bar{A}_x - \bar{A}_x^2$  with  ${}^2\bar{A}_{x+n} - \bar{A}_{x+n}^2$ . Make the same correction to the third line on page 404.

[4/18/2020] On page 1833, in the solution to question 2(b), change  ${}_5|\bar{a}_x$  to  $1000{}_5|\bar{a}_x$ .

[4/18/2020] On page 1834, in the solution to question 2(c)(ii), on the first and fourth lines, change  $v^5 \bar{a}_{x+5}$  to  $1000v^5 \bar{a}_{x+5}$ .

[4/11/2020] On page 28, one line below the table, change  $\text{Var}(99, 98)$  to  $\text{Var}(0.99, 0.98)$ .

[4/6/2020] On page 1391, in the solution to exercise 73.20, on the fourth line, change  $-80$  to  $80$ .

[3/31/2020] On page 1376, two lines from the bottom, change “the each” to “each” (delete “the”).

[3/28/2020] On page 1373, on the second line of the page, change “year  $t$ ” with “year  $k$ ”.

[3/24/2020] On page 1357, one line after equation (72.1), change  $a_B(x, t)$  to  $\ddot{a}_B(x, t)$ .

[3/24/2020] On page 1358, two lines below equation (72.4), change  $(1 + (i^*)^t)$  to  $(1 + i^*)^t$ .

[3/23/2020] On page 1354, in the solution to exercise 71.13, on the fourth line, change “6%” to “5%”.

[3/22/2020] On page 1352, in the solution to exercise 71.2, on the first line, change “calculating” to “calculated”.

[3/21/2020] On page 1342, two lines after the displayed line ending with the highlighted answer 3116.27, change 1.22194 to 2.66952.

[3/20/2020] On page 1342, on the fourth line of the solution to Example 71C, change 94.020.3 to 94.579.7.

[3/13/2020] On page 1325, on the sixth line of the solution to Example 70J, change “retirement” to “withdrawal” twice.

[3/9/2020]

[3/9/2020] On page 1323, on the fifth line, change 17 to 27.

[3/8/2020] On page 1319, on the first line, add “to” before “estimate”.

[3/7/2020] On page 174, 6 lines above Example 10E, change  $m(x, t + k)$  to  $m(x, t + n)$ .

- [3/7/2020] On page 1317, 3 lines above Section 70.1, delete one of the two “benefit” s.
- [2/23/2020] On page 884, on the third line from the bottom, change  $0.05(\bar{A}_{x+10}^{24} - \bar{A}_{x+10}^{24})$  to  $0.05\bar{A}_{x+10}^{24}$
- [2/23/2020] On page 886, four lines from the bottom, change  $p_{x+2}^{(13)}$  to  $p_{x+2}^{13}$ .
- [2/20/2020] On page 884, on the third line, put the subscript  $x$  on  $\bar{A}^{ij}$ :  $\bar{A}_x^{ij}$ .
- [2/16/2020] On page 879, on the second line (expression (47.4)), change  $\ddot{a}_{x+\frac{k+1}{12}:n-\frac{k}{12}}^{(12)\overline{11}}$  to  $\ddot{a}_{x+\frac{k+1}{12}:n-\frac{k+1}{12}}^{(12)\overline{11}}$ .
- [2/16/2020] On page 881, in Example 47B, change the first answer from 110,800 to 104,800.
- [2/13/2020] On page 1148, in the solution to exercise 60.48, on the last line, change  $E[X]$  to  $E[Z]$ .
- [2/10/2020] On page 1122, in exercise 60.6, on the second line, change “annual” to “Annual”.
- [2/9/2020] On page 1111, in the solution to exercise 59.2, on the first line, change “(ii) and (iii)” to “(i) and (ii)”.
- [2/9/2020] On page 1116, on the second line under “Non-independent lives”, add a period after the second integral before “For”.
- [2/4/2020] On page 625, in the solution to exercise 33.5, on the second line from the end, change  $\ddot{a}_{10|}$  to  $\ddot{a}_{17|}$ .
- [1/16/2020] On pages 209–210, in Example 12H and its solution, replace  $A_{\frac{1}{x:2}|}^{(2)}$  with  $A_{\frac{1}{45:2}|}^{(2)}$  three times.
- [1/14/2020] In the alert box on pages 238 and 263, the SOA 283 sample questions no longer exists. The material in this lesson won’t help you much with the current sample questions.
- [1/14/2020] On page 940, in the solution to exercise 48.29, on the third line, change  ${}_k p^{(45)(0.004)}$  to  ${}_k p_{45}^{(\tau)}(0.004)$
- [12/15/2019] On page 460, in the solution to exercise 23.4, 2 lines after the timelines, put a period before “Adding”.
- [12/1/2019] On page 814, 8 lines from the bottom of the page (first line of item 1), change “vary” to “very”.
- [11/28/2019] On page 779, on the last line, change 0.166770 to 0.16770.
- [11/22/2019] On page 725, on the second line, change “time  $t$ ” to “time  $k$ ”.
- [11/18/2019] On page 718, in the solution to Quiz 39-1, on the second line, change  ${}_t q_x$  to  ${}_k | q_x$ .
- [11/6/2019] On page 573, replace the solution to exercise 30.10 with
- The expected present value of benefits and expenses not a percentage of premium is
- $$\begin{aligned} & 1000(A_{40:\overline{20}|} - {}_{20}E_{40}) + 5 + 5\ddot{a}_{40:\overline{20}|} \\ & = 1000(0.38126 - 0.36663) + 10 + 5(12.9935) = 84.5975 \end{aligned}$$
- The expected present value of a payment of 1 per year for 10 years minus 29% in the first year and 9% in the other 9 years is
- $$\begin{aligned} & 0.91\ddot{a}_{40:\overline{10}|} - 0.20 \\ & = 0.91(8.0863) - 0.20 = 7.158533 \end{aligned}$$
- The quotient is  $84.5975/7.158533 = \boxed{11.8177}$ .
- [11/3/2019] On page 531, in the solution to exercise 27.25, on the sixth line, delete the answer key (E). No answer choices are provided for this question.
- [10/31/2019] On page 468, two lines above Example 24A, delete the words “used to”.
- [10/31/2019] On page 1530, in question 6, on the ninth line, change “to not” to “do not”.

- [10/31/2019] On page 1664, in the solution to question 2, on the fifth and seventh lines, replace 23,748 with 24,798.
- [10/31/2019] On page 1673, in the solution to question 6(b), change the final answer from 175,194.6 to 36,584.02.
- [10/31/2019] On page 1685, in the solution to question 6(a), on the first displayed line, change 4,354.70 to 4,152.44. Change the final answer from 21,649.70 to 20,644.16.
- [10/30/2019] On page 508, replace the solution to exercise 26.18 with

The EPV of the special insurance is

$$A_{30:\overline{20}|}^1 + 5(A_{30} - A_{30:\overline{20}|}^1)$$

We calculate this expression. In the Standard Ultimate Life Table,  $A_{30} = 0.07698$ , and

$$\begin{aligned} A_{30:\overline{20}|}^1 &= A_{30:\overline{20}|} - {}_{20}E_{30} \\ &= 0.37900 - 0.37254 = 0.00646 \end{aligned}$$

$$\text{EPV of insurance} = 0.00646 + 5(0.07698 - 0.00646) = 0.35906$$

We calculate the expected present value of a premium annuity of 1 for 20 years, 5 for 15 years afterwards. In the Standard Ultimate Life Table,  $\ddot{a}_{30:\overline{20}|} = 13.0410$ .

$$\begin{aligned} \text{EPV of annuity} &= \ddot{a}_{30:\overline{20}|} + 5(\ddot{a}_{30:\overline{35}|} - \ddot{a}_{30:\overline{20}|}) \\ &= 13.0410 + 5(17.0537 - 13.0410) = 33.1045 \end{aligned}$$

The initial annual net premium is therefore  $0.35906/33.1045 = \boxed{0.01085}$ .

- [10/22/2019] On page 401, on the second line of the second paragraph, change equation (20.2) to equation (20.1).
- [10/22/2019] On page 822, in the solution to exercise 44.9, on the last line, change  ${}_2p_{[x]+2}^{11}$  to  ${}_2p_{[x]+1}^{11}$ .
- [10/19/2019] On page 1580, in the solution to question 3(b), on the third line, change  $100\ddot{a}_{60}$  to  $100\bar{a}_{60}$ .
- [10/17/2019] On page 358, on the first line, change  ${}_{20}E_{85}$  to  ${}_{20}E_{65}$ .
- [10/15/2019] On page 308, in Example 16B(iii), change  $t\%$  to  $5\%$ .
- [10/15/2019] On page 336, in the solution to Example 17A, on the second line change  $q_{47}$  to  $q_{67}$ . On the third line change  $q_{48}$  to  $q_{68}$ .
- [10/12/2019] On page 1345, in both figures, change "Age" to "Service".
- [10/12/2019] On page 1640, in the solution to question 2, 2 lines from the end of the page, insert "Var" before  $(S_n(y_k))$ .
- [10/12/2019] On page 1642, the solution to question 8 has several errors. Replace everything after the third line with:

$$\begin{aligned} {}_{19}E_{51} &= \frac{{}_{20}E_{50}(1.05)}{1 - q_{50}} = \frac{0.34824(1.05)}{1 - 0.001331} = 0.36609 \\ {}_{19}\ddot{a}_{51} &= {}_{19}E_{51} \ddot{a}_{70} = (0.36609)(12.0083) = 4.3962 \\ \pi &= \frac{A_{51}}{\ddot{a}_{51} + {}_{19}\ddot{a}_{51}} = \frac{197.80}{16.8461 + 4.3962} = 9.3116 \end{aligned}$$

Now we calculate the reserve prospectively.

$$\begin{aligned} {}_{10}\ddot{a}_{60} &= {}_{10}E_{60} \ddot{a}_{70} \\ &= (0.57864)(12.0083) = 6.9485 \\ {}_{10}V &= 1000A_{60} - \pi(\ddot{a}_{60} + {}_{10}\ddot{a}_{60}) \\ &= 290.28 - 9.3116(14.9041 + 6.9485) = \boxed{86.7970} \end{aligned}$$

- [10/8/2019] On page 1229, on the fourth line, change “mortality stdy” to “mortality study”.
- [10/8/2019] On page 1267, change the summation index in equation (68.1) to “ $j \mid y_j \leq t$ ”.
- [10/8/2019] On page 1290, four lines below the table, change “(64-10,67-7,s)” to “(64-10,67-2,s)”.
- [10/6/2019] On page 1343, in Example 71D(vi), change “for each year before age 65” to “for each year or fraction of a year before age 65”.
- [10/6/2019] On page 1555, in question 20(iv), change the last sentence to “The benefit is reduced by 8”.
- [10/1/2019] On page 235, in the solution to exercise 12.47, on the fifth line, change the denominator 0.996 to 0.988.
- [9/26/2019] On page 1336, replace the solution to exercise 70.13 with

The number of lives leaving the job in between 2 and 4 years is  $l_{42}^{(\tau)} - l_{44}^{(\tau)} = 152,631.0 - 137,656.1 = 14,974.9$ . Of those,  $i_{42} + i_{43} = 148.8 + 141.3 = 290.1$  leave due to disability. The quotient is  $290.1/14,974.9 =$   
**0.019372**.

- [9/21/2019] On page 93, in exercise 6.36, change  $0 \leq x \leq \omega$  to  $0 \leq t \leq \omega$ .
- [9/21/2019] On page 885, on the third line of Example 47E, change “year death” to “year of death”.
- [9/21/2019] On page 886, on the second line after “If we assume the death benefit is paid...”, the left side of the equation is missing  $1 + i$  and should be  ${}_tV^{(1)}(1 + i)$ .
- [9/21/2019] On page 904, in the solution to exercise 47.3, on the first line, a 36,500 is missing from the left side of the equation and 796.58 should be 756.58; the equation should read  $1.05^{-1/2}(36,500)\ddot{a}_{45:20}^{01} = 1.05^{-1/2}(36,500)(0.02124) = 756.58$ . On the second line, change  $\ddot{a}_{45:20}^{00} + \ddot{a}_{45:20}^{01}$  to  $\ddot{a}_{45:20}^{00} + \ddot{a}_{45:20}^{01}$ .
- [9/10/2019] On pages 868–869, replace the somewhat confusing solution to exercise 46.6 with the following:

Ignoring death, the transition probability matrix for the states low risk, medium risk, and high risk, in that order, is

$$\begin{pmatrix} 0.6 & 0.4 & 0 \\ 0.2 & 0.5 & 0.3 \\ 0 & 0.2 & 0.8 \end{pmatrix}$$

The probability of death is 0.05. Making death the fourth state, we multiply our  $3 \times 3$  matrix by 0.95 and put 0.05 in the fourth column to represent the probability of transition to the death state to obtain the following transition probability matrix:

$$\begin{pmatrix} 0.57 & 0.38 & 0 & 0.05 \\ 0.19 & 0.475 & 0.285 & 0.05 \\ 0 & 0.19 & 0.76 & 0.05 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

With this transition matrix, we can compute the state vector for each year. At the beginning of the first year, the system is in state 1. At the beginning of the second year, the state vector is  $(0.57 \ 0.38 \ 0 \ 0.05)$ . At the beginning of the third year, the state vector is

$$(0.57 \ 0.38 \ 0 \ 0.05) \begin{pmatrix} 0.57 & 0.38 & 0 & 0.05 \\ 0.19 & 0.475 & 0.285 & 0.05 \\ 0 & 0.19 & 0.76 & 0.05 \\ 0 & 0 & 0 & 1 \end{pmatrix} = (0.3971 \ 0.3971 \ 0.1083 \ 0.0975)$$

Now we compute expected present value of losses. In each year, we multiply the probability of each start-of-year state by 0.95, since only survivors result in paid losses, and by 100, 300, or 1000 for states

1, 2, 3 respectively, and discount for the number of years at 6%. In year 1, we get  $100(0.95)/1.06 = 89.62$ . In year 2, we get

$$\frac{(0.57(100) + 0.38(300))(0.95)}{1.06^2} = 144.58$$

In year 3, we get

$$\frac{(0.3971(100) + 0.3971(300) + 0.1083(1000))(0.95)}{1.06^3} = 213.08$$

The expected present value of losses over a three year period is  $89.62 + 144.58 + 213.08 = \boxed{447.28}$ .

[9/10/2019] On page 1321, one line above Example 70G, change “ $d_x^{(j)}$ -type” to “ $d_x^{(j)}$ -type”.

[9/10/2019] On page 1358.7, in equations (72.4) and (72.5), change the summation index from  $i$  to  $t$ :  $\sum_{t=0}^{65-x}$ .

[9/10/2019] On page 1361, on the line below equation (72.3), add a  $-1$ :

$$\text{where } i^* = \frac{1+i}{c(1+j)} - 1.$$

In equations (72.4) and (72.5), change the summation index from  $i$  to  $t$ :  $\sum_{t=0}^{65-x}$

[9/10/2019] On page 1375, on the second line below the heading **DPP**, change  $=$  to  $\geq$ :

$$\sum_{k=0}^m \Pi_k v^k \geq 0$$

[9/10/2019] On page 1402, in the solution to exercise 74.8, 3 lines from the end, change  $(15 - 10)$  to  $(10 - 15)$ .

[8/30/2019] On page 1556, in question 2(b)(ii), change “healthy” to “sick”.

[8/9/2019] On page 188, in the solution to exercise 10.17, on the second line, change  $0.01Z_{2017}^{(2)}$  to  $0.05Z_{2017}^{(2)}$ .

[8/7/2019] On page 379, in the solution to exercise 19.39, replace the last two lines with

$$\begin{aligned} {}_{30|}\ddot{a}_{30} &= {}_{30}E_{30} \ddot{a}_{60} = (0.22420)(14.9041) = 3.3415 \\ P &= \frac{200 {}_{30|}\ddot{a}_{30}}{1 - A_{30:\overline{30}|}^1} = \frac{(200)(3.3415)}{1 - 0.01190} = \boxed{676.35} \end{aligned}$$

[5/24/2019] On page 160, in the solution to exercise 9.17, on the first line, change  $l_{80}$  to  $l_{[80]}$ .

[5/24/2019] On page 478, in the solution to exercise 24.13, this solution is based on the old Illustrative Life Tables, which has interest functions at 6%. The current Standard Ultimate Life Table gives interest functions at 5%, so you would have to calculate all interest functions from first principles if you did not have the Illustrative Life Tables available.

[5/24/2019] On page 1026, in the solution to exercise 53.17, on the second line, delete the first left parenthesis in the first exponent.

[3/27/2019] On page 1369, on the last line of the fourth paragraph, change “higher of lower” to “higher or lower”.

[7/23/1970] On page 1698, in the solution to question 4, on the second-to-last line, change  $e_{68}$  to  $e_{68}^2$ .