Errata and Updates for ASM Exam ASTAM (Third Edition Second Printing) Sorted by Date

[9/7/2026] On page 736, six lines above Example 40D, change the first $\hat{f}_j$ to $\bar{f}_j$.

[9/7/2026] On page 749, in the solution to exercise 40.5, change "There are 6 observations" to "There are 5 observations". And change the final answer from 4 to 3.

[9/7/2026] On page 749, in the solution to exercise 40.6, on the second-to-last line, change $f_{0,3}$ to $f_{3,0}$

[5/6/2026] On page 154, on the second line from the bottom, change the numerator of the fraction, 0.95(100), to 0.05(100). On the last line of the page, change the numerator of the fraction, 0.995(100), to 0.005(100).

[4/20/2026] On page 817.1, in question 1(b), change LAS/100,000 to $\text{LAS}^2/100{,}000$.

[4/15/2026] On pages 776–779, the solutions to 42.1, 42.2, 42.3, 42.4, 42.6, and 42.9 use an incorrect formula for σ_j^2. The corrected solutions are shown on the pages at the end of this errata listing.

42.9. You are given the following loss development data:

Cumulative payments				
	Development Year			
	0	1	2	3
0	2000	3000	3200	3360
1	2200	2700	3070	
2	2400	3540		
3	2800			

Losses are fully developed at the end of the development year 3.

Using Mack's method, calculate the covariance of the estimation errors,

$$\sum_{i=1}^{I-1}\sum_{k=i+1}^{I}\left(\hat{C}_{i,J}-\mathbf{E}[C_{i,J}\mid\mathcal{D}_I]\right)\left(\hat{C}_{k,J}-\mathbf{E}[C_{k,J}\mid\mathcal{D}_I]\right)$$

Additional old SOA Exam ASTAM questions: S23:6c,6d,6e, F23:6, F24:6c,6d,6e

Solutions

42.1. The link factor $\hat{f}_2$ is

$$\hat{f}_2=\frac{236+275}{225+262}=1.049281$$

Then

$$\sigma_2^2=225\left(\frac{236}{225}-1.049281\right)^2+262\left(\frac{275}{262}-1.049281\right)^2=6.4406\times 10^{-5}$$

Then $\sigma_2=\sqrt{6.4406\times 10^{-5}}=\boxed{0.00802531}$.

42.2. We back out $C_{2,1}$.

$$\begin{aligned}1.16=\hat{f}_0=\frac{1832+2232+C_{2,1}+2700}{1558+1892+2100+2400}&=\frac{6764+C_{2,1}}{7950}\\ 6764+C_{2,1}&=9222\\ C_{2,1}&=2458\end{aligned}$$

Then

$$\begin{aligned}\hat{f}_1&=\frac{2000+2415+2617}{1832+2232+2458}=1.078197\\ \sigma_1^2&=\frac{1}{2}\left(1832\left(\frac{2000}{1832}-1.078197\right)^2+2232\left(\frac{2415}{2232}-1.078197\right)^2+2458\left(\frac{2617}{2458}-1.078197\right)^2\right)\\ &=\boxed{0.407466}\end{aligned}$$

42.3. We first need $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$.

$$\begin{aligned}\hat{f}_1&=\frac{1060+1190+1499}{1020+1040+1200}=1.15\\ \hat{\sigma}_1^2&=\frac{1}{2}\left(1020\left(\frac{1060}{1020}-1.15\right)^2+1040\left(\frac{1190}{1040}-1.15\right)^2+1200\left(\frac{1499}{1200}-1.15\right)^2\right)=12.17704\end{aligned}$$

$$\hat{f}_2 = \frac{1100 + 1375}{1060 + 1190} = 1.1$$
$$\hat{\sigma}_2^2 = 1060\left(\frac{1100}{1060} - 1.1\right)^2 + 1100\left(\frac{1375}{1190} - 1.1\right)^2 = 7.769938$$

The answer is
$$\min(7.769938, 12.17704, 7.769938^2/12.17704) = \boxed{\mathbf{4.957851}}$$

42.4.

(a) By the assumptions of Mack's method, $\mathrm{Var}(C_{3,2} \mid C_{3,1}) = C_{3,1}\sigma_1^2$. We calculate $\hat{\sigma}_1^2$.

$$\hat{f}_1 = \frac{510 + 500 + 694}{430 + 490 + 500} = 1.2$$
$$\hat{\sigma}_1^2 = \frac{1}{2}\left(430\left(\frac{510}{430} - 1.2\right)^2 + 490\left(\frac{500}{490} - 1.2\right)^2 + 500\left(\frac{694}{500} - 1.2\right)^2\right) = 16.77990$$
$$\mathrm{Var}(C_{3,2} \mid C_{3,1}) = 755(16.77990) = \boxed{\mathbf{12{,}668.83}}$$

(b) We use conditional variance, conditioning on $C_{3,2} \mid C_{3,1}$.

$$\begin{aligned}\mathrm{Var}(C_{3,3} \mid C(3,1)) &= \mathbf{E}\left[\mathrm{Var}(C_{3,3} \mid C_{3,2})\right] + \mathrm{Var}\left(\mathbf{E}[C_{3,3} \mid C_{3,2}]\right)\\ &= \mathbf{E}[(C_{3,2} \mid C_{3,1})\sigma_2^2] + \mathrm{Var}((C_{3,2} \mid C_{3,1})f_2)\\ &= C_{3,1}f_1\sigma_2^2 + f_2^2 C_{3,1}\sigma_1^2\end{aligned}$$

We calculate $\hat{f}_2$ and $\hat{\sigma}_2^2$.

$$\hat{f}_2 = \frac{530 + 500}{510 + 500} = 1.019802$$
$$\hat{\sigma}_2^2 = 510\left(\frac{530}{510} - 1.019802\right)^2 + 500(1 - 1.019802)^2 = 0.388274$$
$$\mathrm{Var}(C_{3,3} \mid C_{3,1}) = 755\big(1.2(0.388274) + 1.019802^2(16.77990)\big) = 755(17.91696) = \boxed{\mathbf{13{,}527.31}}$$

We can also compute the variance using formula (42.5) setting $J = 3$:

$$\hat{C}_{3,2} = 755(1.2) = 906$$
$$\hat{C}_{3,3} = 906(1.019802) = 923.9406$$
$$\mathrm{Var}(C_{3,3} \mid C_{3,1}) = 923.9406^2\left(\frac{16.77990}{1.2^2(755)} + \frac{0.388274}{1.019802^2(906)}\right) = \boxed{\mathbf{13{,}527.31}}$$

42.5. We will use formula (42.6). We first calculate $\hat{C}_{3,j}$ for $j > 1$.

$$\hat{C}_{3,2} = 3250(1.078) = 3503.5$$
$$\hat{C}_{3,3} = 3503.5(1.048) = 3671.668$$
$$\hat{C}_{3,4} = 3871.668(1.043) = 3829.550$$

Now we have everything we need for formula (42.6).

$$\mathrm{Var}(C_{3,4} \mid C_{3,1}) = 3829.550^2\left(\frac{0.00019}{1.078^2(3250)} + \frac{0.000093}{1.048^2(3503.5)} + \frac{0.000048}{1.043^2(3671.668)}\right) = \boxed{\mathbf{1.2685}}$$

42.6. We will use formula (42.6).

$$\hat{f}_0 = \frac{209+210+300}{184+205+215} = 1.190397$$
$$\hat{f}_1 = \frac{222+255}{209+210} = 1.138425$$
$$\hat{f}_2 = \frac{240}{222} = 1.081081$$
$$\hat{\sigma}_0^2 = \frac{1}{2}\left(184\left(\frac{209}{184} - 1.190397\right)^2 + 203\left(\frac{210}{200} - 1.190397\right)^2 + 215\left(\frac{300}{215} - 1.190397\right)^2\right) = 7.613823$$
$$\hat{\sigma}_1^2 = 209\left(\frac{222}{209} - 1.138425\right)^2 + 210\left(\frac{255}{210} - 1.138425\right)^2 = 2.422830$$
$$\hat{\sigma}_2^2 = \min\left(2.422830, 7.613823, 2.422830^2/7.613823\right) = 0.770980$$

For AY3,

$$\hat{C}_{3,1} = 220(1.190397) = 261.89$$
$$\hat{C}_{3,2} = 261.89(1.138425) = 298.14$$
$$\hat{C}_{3,3} = 298.14(1.081081) = 322.31$$

Using formula (42.6),

$$\operatorname{Var}(C_{3,3} \mid C_{3,0}) = 322.31^2\left(\frac{7.613823}{1.190397^2(220)} + \frac{2.422830}{1.138425^2(261.89)} + \frac{0.770980}{1.081081^2(298.14)}\right) = \boxed{3508.609}$$

42.7. We'll use equation (42.7). The sum has only one term since $I-1 = J-1$, making the formula

$$\left(\hat{C}_{1,9} - \mathbf{E}[C_{1,9} \mid \mathcal{D}_9]\right)^2 = \frac{\hat{C}_{1,9}^2\sigma_8^2}{\hat{f}_8^2 S_8}$$

$S_8 = \hat{C}_{0,8} = 17{,}800$. Also

$$\hat{C}_{1,9} = C_{1,8}\hat{f}_8$$

so the formula becomes

$$\left(\hat{C}_{1,9} - \mathbf{E}[C_{1,9} \mid \mathcal{D}_9]\right)^2 = \frac{C_{1,8}^2\hat{f}_8^2\sigma_8^2}{\hat{f}_8^2(17{,}800)} = \frac{C_{1,8}^2\sigma_8^2}{17{,}800}$$

since the $\hat{f}_8^2$s in the equation cancel. Thus we get

$$\left(\hat{C}_{1,9} - \mathbf{E}[C_{1,9} \mid \mathcal{D}_9]\right)^2 = 19{,}500^2\left(\frac{0.0036}{17{,}800}\right) = \boxed{76.9045}$$

42.8. First we compute $\hat{C}_{3,4}$ and S_j, $j = 1, 2, 3$.

$$\hat{C}_{3,4} = C_{3,1}\hat{f}_2\hat{f}_3\hat{f}_4 = 1660(1.15)(1.1)(1.01) = 2120.899$$
$$S_1 = 1020 + 1040 + 1200 = 3260$$
$$S_2 = 1060 + 1190 = 2250$$
$$S_3 = 1100$$

Now we use formula (42.7).

$$\left(\hat{C}_{3,4} - \mathbf{E}[C_{3,1} \mid \mathcal{D}_4]\right)^2 = 2120.899^2\left(\frac{0.011070}{1.15^2(3260)} + \frac{0.006953}{1.1^2(0.006953)} + \frac{0.004367}{1.01^2(2250)}\right) = \boxed{40.5434}$$

42.9. First calculate the $\hat{f}_j$s, the $\hat{\sigma}_j^2$s, and all projected $\hat{C}_{i,j}$s.

$$\begin{aligned}
S_1 &= 3000 + 2700 = 5700\\
\hat{f}_1 &= \frac{3200 + 3070}{5700} = 1.1\\
\hat{f}_2 &= \frac{3360}{3070} = 1.05
\end{aligned}$$

Using these $\hat{f}_j$s, the projected values, shown in italics, are:

Cumulative payments				
	Development Year			
	0	1	2	3
0	2000	3000	3200	3360
1	2200	2700	3070	*3223.5*
2	2400	3540	*3894*	*4088.7*
3	2800	*3920*	*4312*	*4527.6*

$$\begin{aligned}
\hat{\sigma}_0^2 &= \frac{1}{2}\left(2000(1.5-1.4)^2 + 2200\left(\frac{2700}{2200} - 1.4\right)^2 + 2400\left(\frac{3540}{2400} - 1.4\right)^2\right) = 49.56818\\
\hat{\sigma}_1^2 &= 3000\left(\frac{3200^{\,2}}{3000} - 1.1\right)^2 + 2700\left(\frac{3070}{2700} - 1.1\right)^2 = 7.037037\\
\hat{\sigma}_2^2 &= \frac{7.037037^2}{49.56818} = 0.999026
\end{aligned}$$

Now we have everything we need to calculate the last term of formula (42.9)

$$\begin{aligned}
&\sum_{i=1}^{I-1}\sum_{k=i+1}^{I}\left(\hat{C}_{i,J} - \mathbf{E}[C_{i,J} \mid \mathcal{D}_I]\right)\left(\hat{C}_{k,J} - \mathbf{E}[C_{k,J} \mid \mathcal{D}_I]\right)\\
&\quad = 3223.5\left(\frac{0.999026}{1.05^2(3200)}\right)(4088.7 + 4527.6) + 4088.7\left(\frac{7.037037}{1.1^2(5700)} + \frac{0.999026}{1.05^2(3200)}\right)(4527.6)\\
&\quad = 7{,}864.96 + 24{,}129.92 = \boxed{\mathbf{31{,}994.88}}
\end{aligned}$$